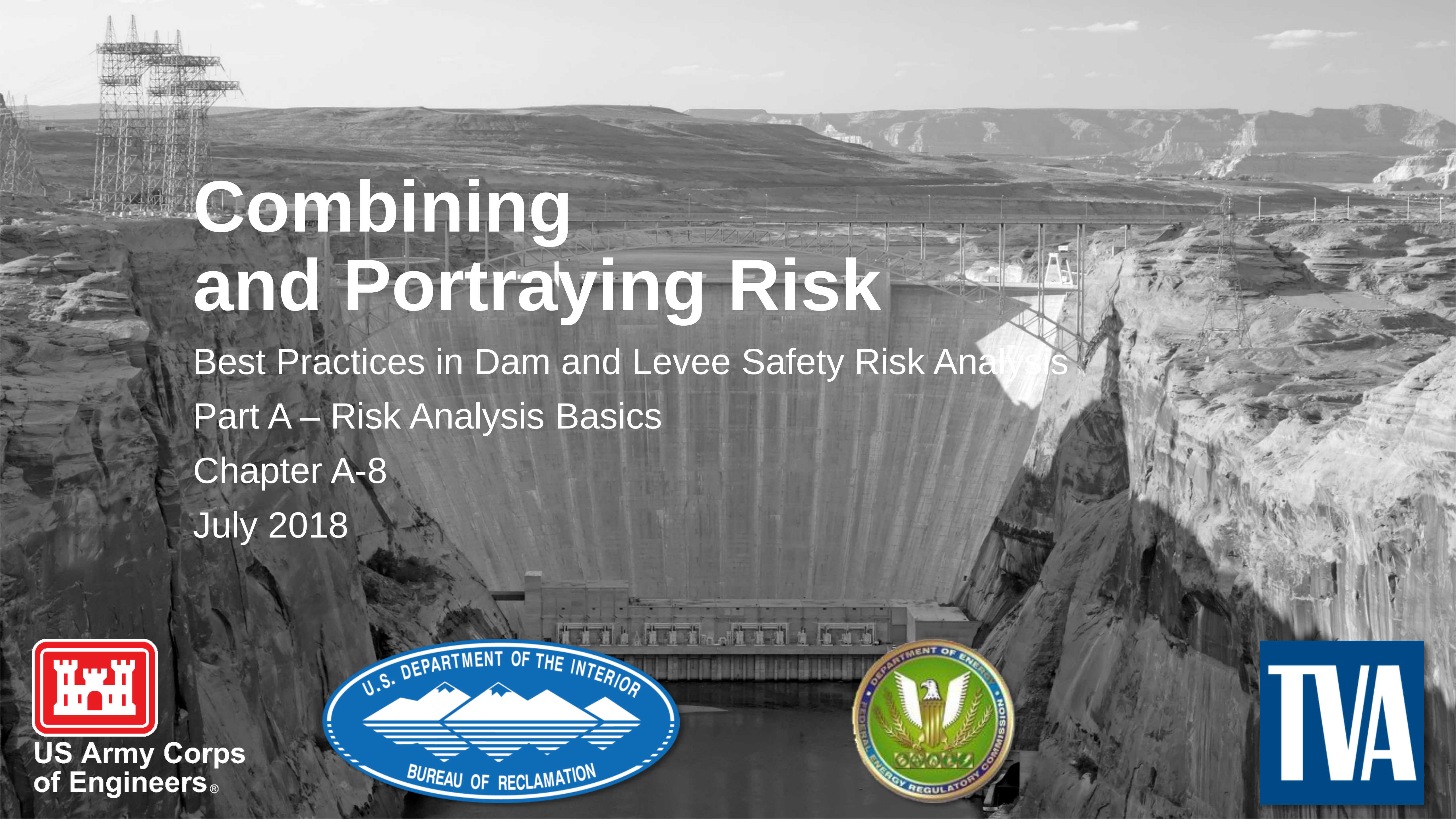


# Combining and Portraying Risk

Best Practices in Dam and Levee Safety Risk Analysis

Part A – Risk Analysis Basics

Chapter A-8

July 2018

US Army Corps of Engineers®

U.S. Department of the Interior Bureau of Reclamation

Department of Energy Federal Energy Regulatory Commission

TVA

# Key Concepts

- Risk estimates need to be **combined** in an informed manner so that the collective impact can be **portrayed** in a way that can be effectively communicated to decision makers and used to take appropriate action
- Concepts include:
  - fN versus FN diagrams
  - Double counting of the intersection and methods to address it
  - Input versus output uncertainty

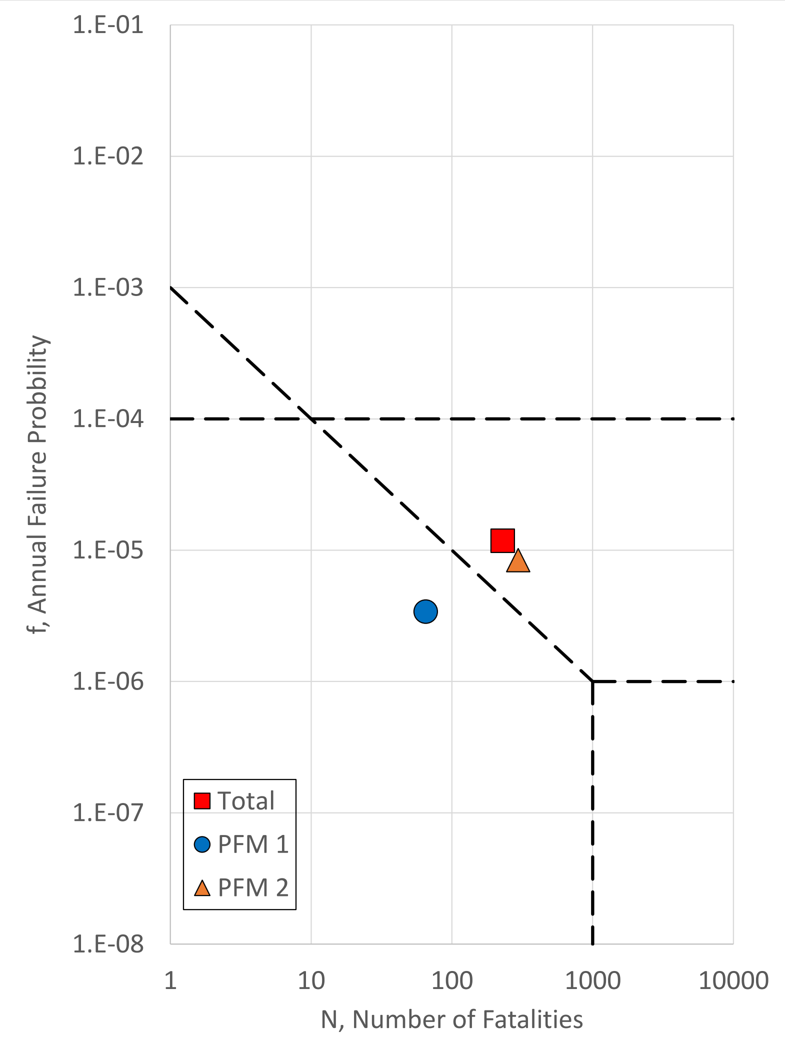
# Basic Problem

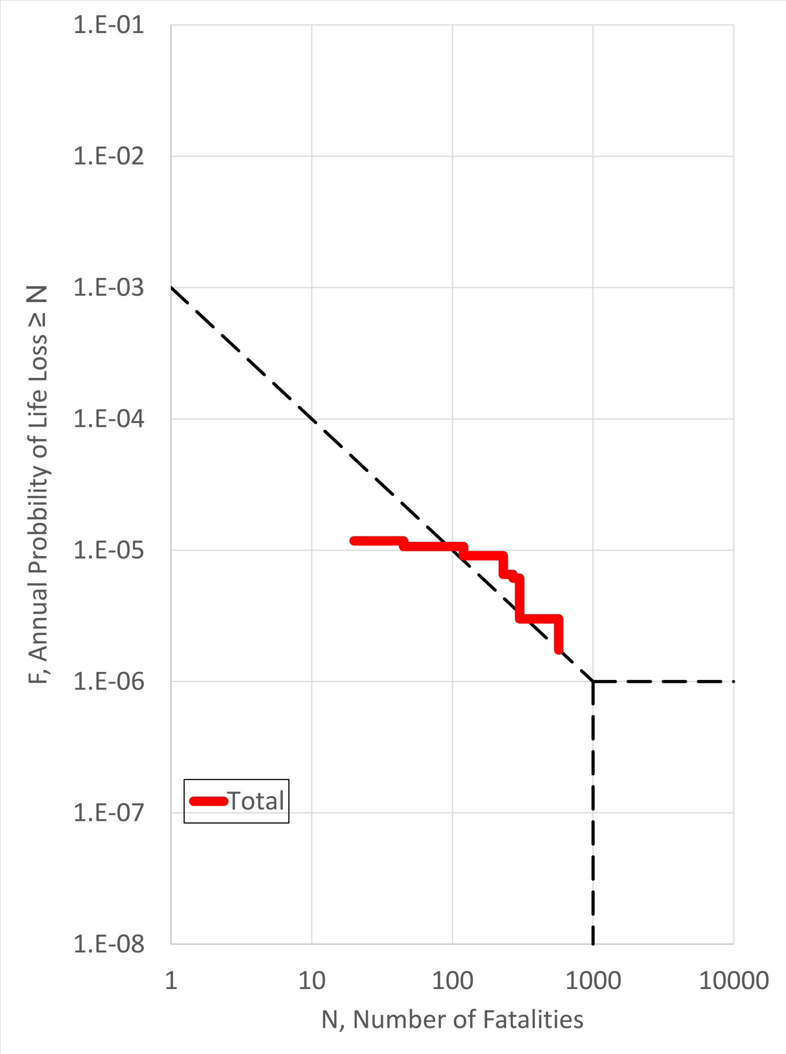
- The total AFP for a facility is equal to the union of the probability of the individual PFMs
- The formula for the union probability of two events is:
  $P(A \cup B) = P(A) + P(B) - P(AB)$
- The total AFP is typically calculated as the simple sum of the individual PFM risk estimates
- Whether this is a concern, and to what extent, depends on the specific situation under consideration as well as the broader viewpoint of those performing the risk analysis

# Two schools of thought

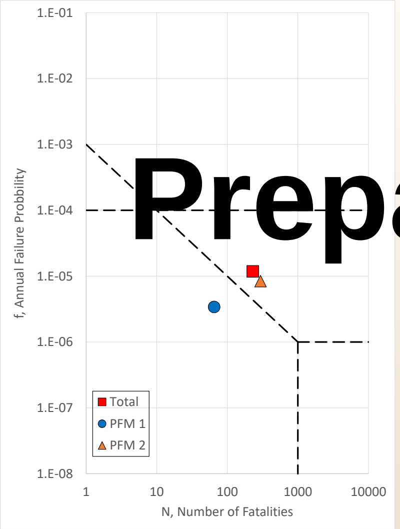
- Viewpoint 1
  - The basic unit of meaning is the individual PFM. PFMs should be developed independently, without up-front consideration of how other PFMs might be affected, and the focus of the risk analysis should be on the PFMs that plausibly control the risk of failure
  - fN chart is the preferred presentation format
- Viewpoint 2
  - The basic unit of meaning is the facility-wide event tree. The nature of the relationship between the PFMs is determined by the fact that, logically, they should all be able to fit into such an event tree without violating the basic axioms of probability theory
  - FN chart is the preferred presentation format

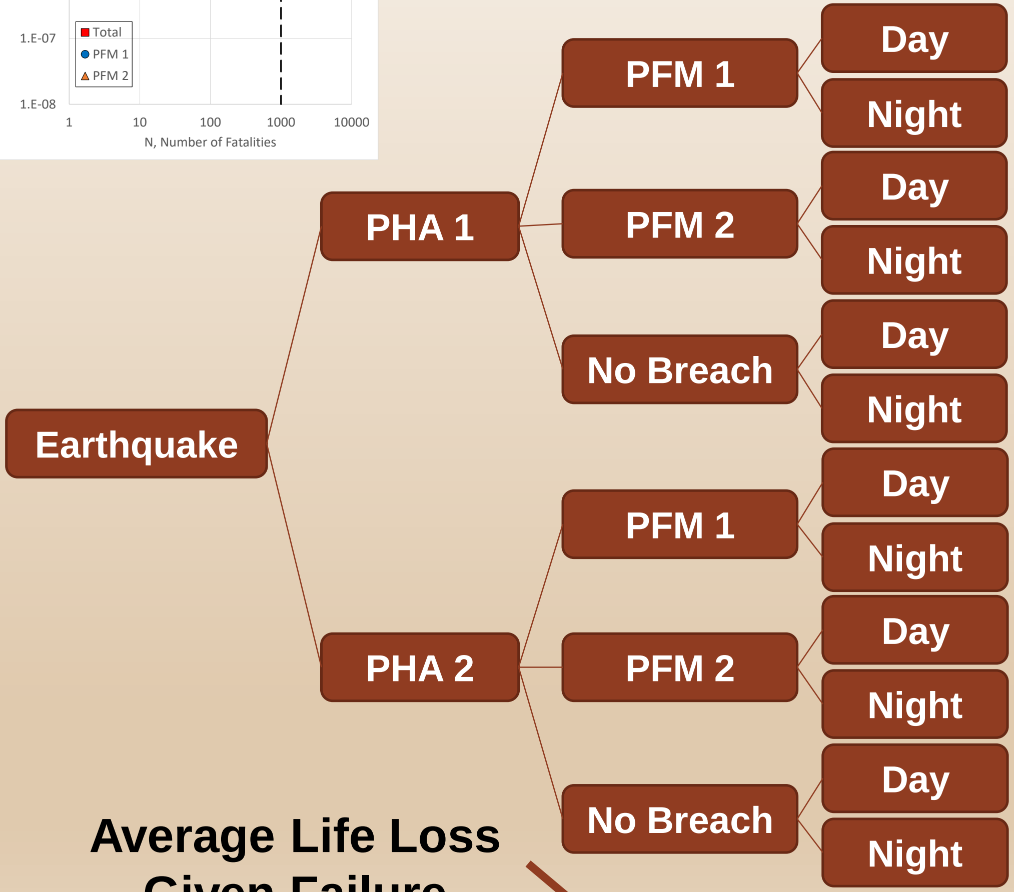
# fN chart versus FN chart



# Preparing Data for an f,N Risk Plot



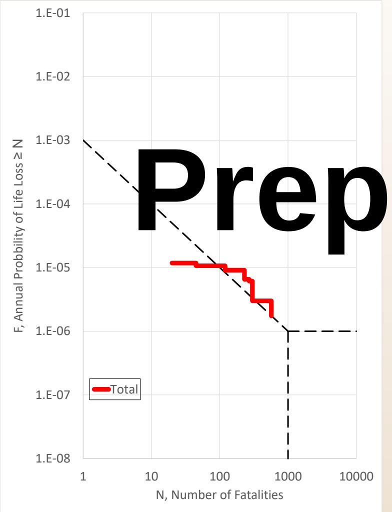

| APF (f) | N | ALL (f * N) |
|---|---|---|
| 1.1E-6 | 20 | 2.3E-5 |
| 1.6E-6 | 45 | 7.0E-5 |
| 2.3E-6 | 120 | 2.7E-4 |
| 3.1E-6 | 270 | 8.5E-4 |
| 3.8E-3 | 5 | 1.9E-2 |
| 5.2E-3 | 8 | 4.2E-2 |
| 2.9E-7 | 120 | 3.5E-5 |
| 4.1E-7 | 230 | 9.3E-5 |
| 1.3E-6 | 300 | 3.8E-4 |
| 1.7E-6 | 570 | 9.9E-4 |
| 4.2E-4 | 30 | 1.3E-2 |
| 5.8E-4 | 70 | 4.0E-2 |

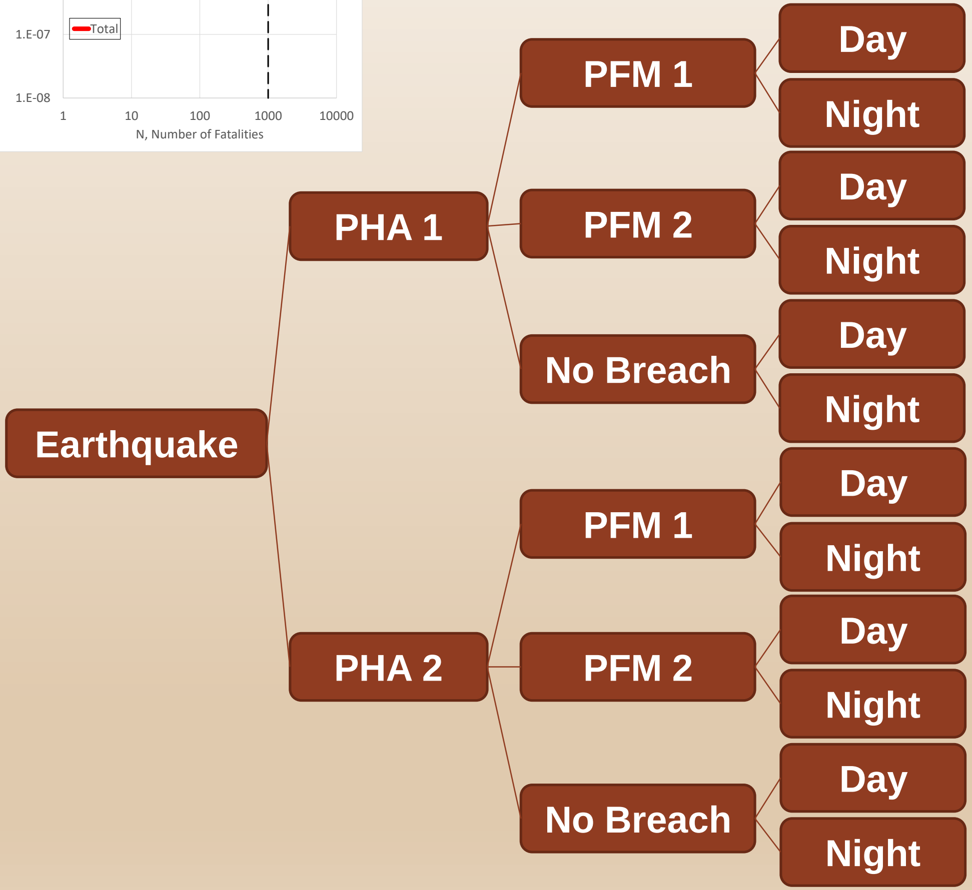
| APF (f) | N | ALL (f * N) |
|---|---|---|
| 1.1E-6 | 20 | 2.3E-5 |
| 1.6E-6 | 45 | 7.0E-5 |
| | | |
| | | |
| | | |
| | | |
| 2.9E-7 | 120 | 3.5E-5 |
| 4.1E-7 | 230 | 9.3E-5 |
| | | |
| | | |
| | | |
| | | |
| ∑ = 3.4E-6 | 65 | ∑ = 2.2E-4 |

**Average Life Loss Given Failure**

$$\bar{N} = E[N|PFM1] = \frac{\sum f_i N_i}{\sum f_i} = \frac{2.2\ x\ 10^{-4}}{3.4\ x\ 10^{-6}} = 65$$

# Preparing Data for an F,N Risk Plot



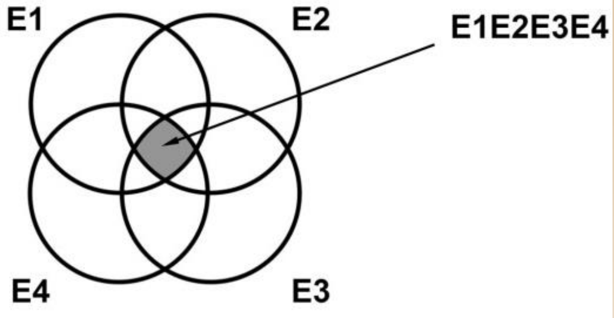

| APF (f) | N | Rank | |
|---|---|---|---|
| 1.1E-6 | 20 | 8 | Lowest N |
| 1.6E-6 | 45 | 7 | |
| 2.3E-6 | 120 | 5 | |
| 3.1E-6 | 270 | 3 | |
| 3.8E-3 | 5 | | |
| 5.2E-3 | 8 | | |
| 2.9E-7 | 120 | 6 | |
| 4.1E-7 | 230 | 4 | |
| 1.3E-6 | 300 | 2 | |
| 1.7E-6 | 570 | 1 | Highest N |
| 4.2E-4 | 30 | | |
| 5.8E-4 | 70 | | |

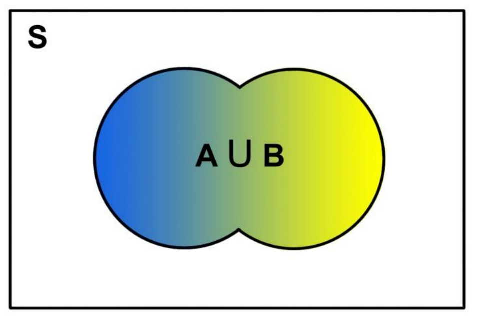
| Rank | f | N | F |
|---|---|---|---|
| 1 | 1.7E-6 | 570 | 1.7E-6 |
| 2 | 1.3E-6 | 300 | 3.0E-6 |
| 3 | 3.1E-6 | 270 | 6.1E-6 |
| 4 | 4.1E-7 | 230 | 6.5E-6 |
| 5 | 2.3E-6 | 120 | |
| 6 | 2.9E-7 | 120 | 9.1E-6 |
| 7 | 1.6E-6 | 45 | 1.1E-5 |
| 8 | 1.1E-6 | 20 | 1.2E-5 |

$$F(N \geq 270) = f(N = 570) + f(N = 300) + f(N = 270) = 1.7E\text{-}6 + 1.3E\text{-}6 + 3.1E\text{-}6 = 6.1E\text{-}6$$

# Basic Problem (Viewpoint 1)

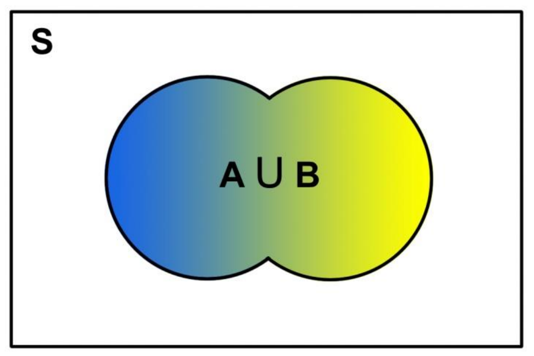
- Dam failure by an individual PFM is defined as the occurrence of all the events of the PFM sequence
- The PFM failure event is therefore an intersection event ($E_1$ AND $E_2$ AND $E_3$ AND $E_4$)
- However, multiple PFMs will typically apply at a dam
- For the dam as a whole, the occurrence of <u>any</u> of the *n* PFMs would result in failure. The failure event for the dam is therefore a union event (fails by $PFM_A$ OR fails by $PFM_B$ ...)

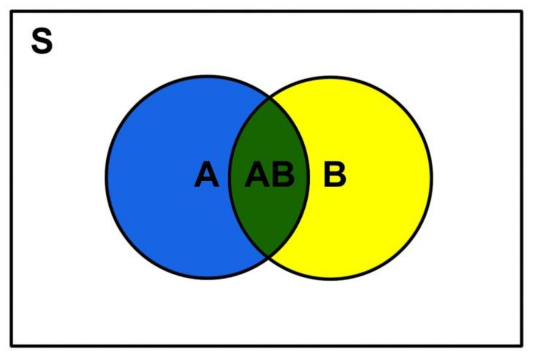


# Basic Problem (Viewpoint 1)

- The probability of the union of two events A and B is given by P(A U B) = P(A) + P(B) – P(AB)
- However, the way that the total AFP is usually calculated is as the simple sum of the individual PFM AFPs
- It is a **fact** that this approach results in over counting of the intersection
- In many cases, the over counting will have **little impact** on the overall dam safety case. However, it is important to recognize that this is an *assumption* that may not be equally appropriate in all cases



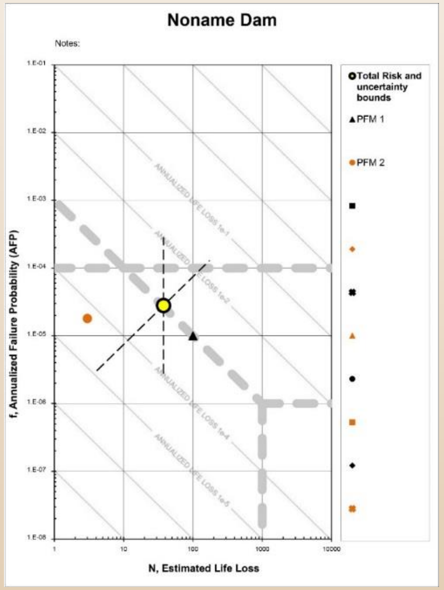

Instructions for the fN chart data table: **Type only within the red borders** ; Enter the name and type (static, hydro, seismic...) of the failure mode; Inlcude only the ten most critical failure modes; If there are less than ten failure modes, leave the extra "PFM name and type" fields blank; Enter dam name both on chart and to the right.

Navajo Dam 2016 CR

| PFM name and type | AFP Low | AFP mean | AFP high | Life Loss Low (> 0) | Life Loss Mean (≥ 1) | Life Loss High | Annualized Life Loss Low | Annualized Life Loss Mean | Annualized Life Loss High |
|---|---|---|---|---|---|---|---|---|---|
| PFM 1. Internal erosion into or along the right abutment | 2.20E-06 | 2.20E-05 | 2.20E-04 | 446 | 554 | 669 | 9.79E-04 | 1.22E-02 | 1.47E-01 |
| PFM 2. Internal erosion of the right abutment foundation | 6.33E-06 | 6.33E-05 | 6.33E-04 | 305 | 374 | 448 | 1.93E-03 | 2.37E-02 | 2.84E-01 |
| PFM 3. Internal erosion into or along the left abutment | 3.68E-08 | 3.68E-07 | 3.68E-06 | 446 | 554 | 669 | 1.64E-05 | 2.04E-04 | 2.46E-03 |
| PFM 4. Internal erosion due to hydraulic fracturing | 6.86E-08 | 6.86E-07 | 6.86E-06 | 446 | 554 | 669 | 3.06E-05 | 3.80E-04 | 4.59E-03 |
| PFM H1: Internal erosion of the crest under flood conditions | 3.09E-07 | 3.09E-06 | 3.09E-05 | 214 | 253 | 293 | 6.61E-05 | 7.81E-04 | 9.04E-03 |
| PFM EQ1: Seismic Liq OT or Cracking | 7.35E-07 | 7.35E-06 | 7.35E-05 | 1329 | 1743 | 2176 | 9.77E-04 | 1.28E-02 | 1.60E-01 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| **Total Risk and uncertainty bounds** | 9.67E-06 | **9.67E-05** | 9.67E-04 | (Life Loss weighted mean) | 516.98 | (Life Loss weighted mean) | 4.00E-03 | **5.00E-02** | 6.06E-01 |

# Option 1: Ignore the Intersection Event and its Probability

- Consider a pair of PFMs with statistically independent response events A and B (e.g. A = earth embankment fails, B = concrete spillway fails)
- Assume the conditional probability of A (given the occurrence of a 50,000-year quake Q) is 0.5 and the probability of B is 0.9
- Uncorrected total AFP is P(AQ) + P(BQ) = 1.4/50,000
- At the response level, the probability of the union of A and B is given by P(A U B | Q) = P(A | Q) + P(B | Q) – P(AB | Q) = 0.5 + 0.9 – 0.45 = 0.95
- Corrected total seismic AFP = P(AQ U BQ) = 0.95/50,000
- 30 percent reduction seems significant – but is it really?



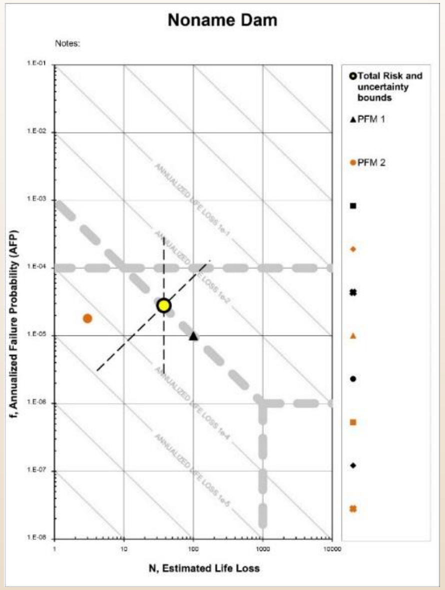
# Option 2: Adjust the total AFP

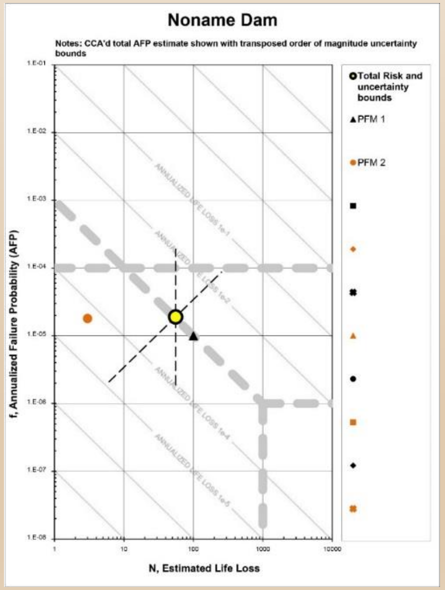
- The total AFP would be adjusted directly on the fN chart
- The individual PFM risk estimates would not be adjusted
- For a discussion of ALL adjustments, see written chapter

*Instructions for the fN chart data table:* ***Type only within the red borders*** *; Enter the name and type (static, hydro, seismic...) of the failure mode; Inlcude only the ten most critical failure modes; If there are less than ten failure modes, leave the extra "PFM name and type" fields blank; Enter dam name both on chart and to the right.*

Noname Dam

| PFM name and type | AFP Low | AFP mean | AFP high | Life Loss Low (> 0) | Life Loss Mean (≥ 1) | Life Loss High | Annualized Life Loss Low | Annualized Life Loss Mean | Annualized Life Loss High |
|---|---|---|---|---|---|---|---|---|---|
| PFM 1 | | 1.00E-05 | | | 100 | | 0.00E+00 | 1.00E-03 | 0.00E+00 |
| PFM 2 | | 1.80E-05 | | | 3 | | 0.00E+00 | 5.40E-05 | 0.00E+00 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| Note: CCA applied to total AFP at the conditional failure probability level | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| Note: Corrected total AFP; individual PFM AFPs will not sum to reported total AFP | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| | | | | | | | 0.00E+00 | 0.00E+00 | 0.00E+00 |
| **Total Risk and uncertainty bounds** | 0.00E+00 | **1.90E-05** | 0.00E+00 | (Life Loss weighted mean) | 55.47 | (Life Loss weighted mean) | 0.00E+00 | **1.05E-03** | 0.00E+00 |

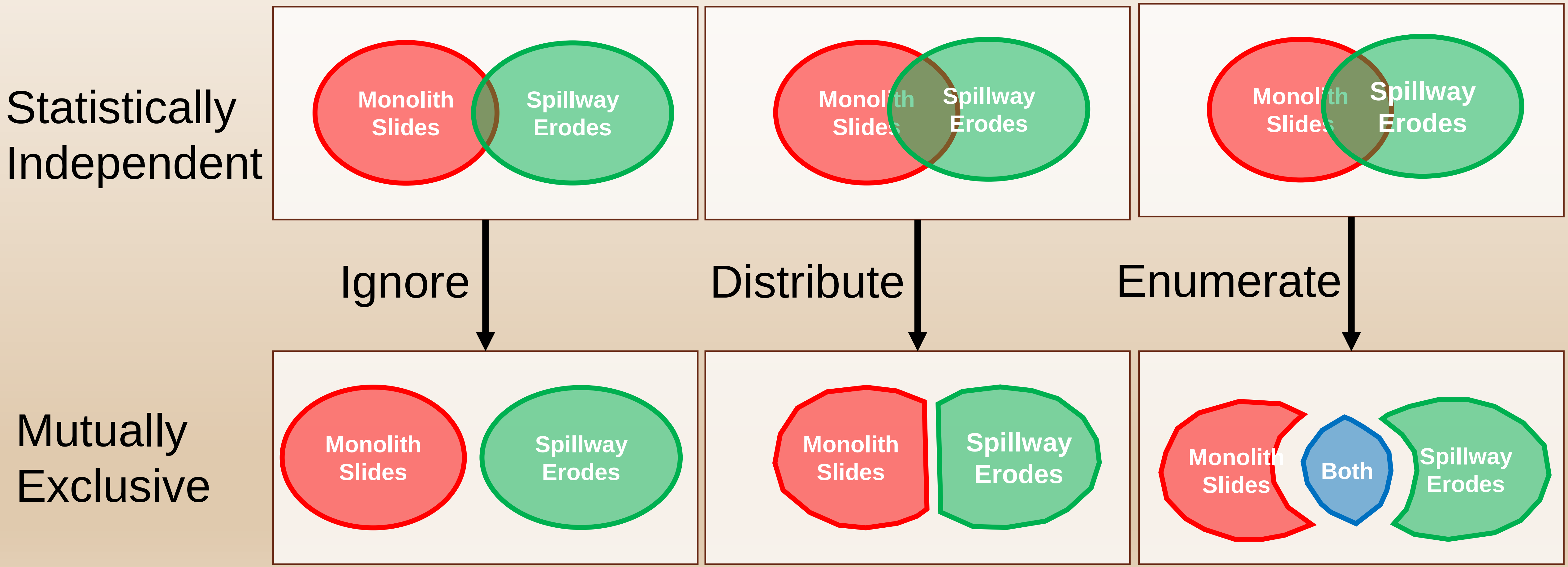


# Combining System Response Probabilities (Viewpoint 2)

- PFMs are typically assumed to be statistically independent
  - Simplifies the probability estimation for each PFM
  - Using de Morgan's rule, $P(total) = 1 - \prod_{i=1}^{n}(1 - p_i)$
- Event tree branches must be mutually exclusive so they can be summed
  - $P(total) = \sum_{i=1}^{n} p_i$
- Risks are typically attributed and portrayed by individual PFMs
- Options to make it work
  - Ignore the intersection events and their probability
  - Ignore the intersection events and distribute their probability
  - Enumerate the intersection events and their probability
- Any adjustments should be made to each loading partition

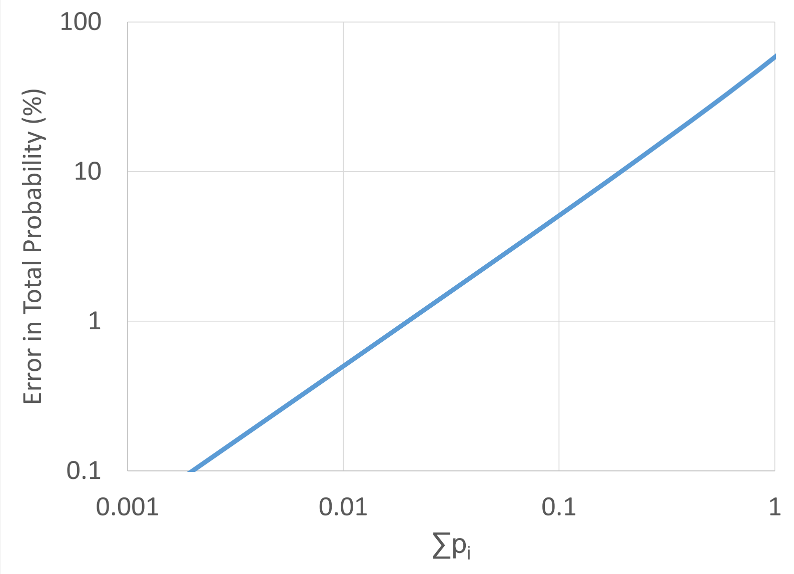
# Options for Combining System Response Probabilities



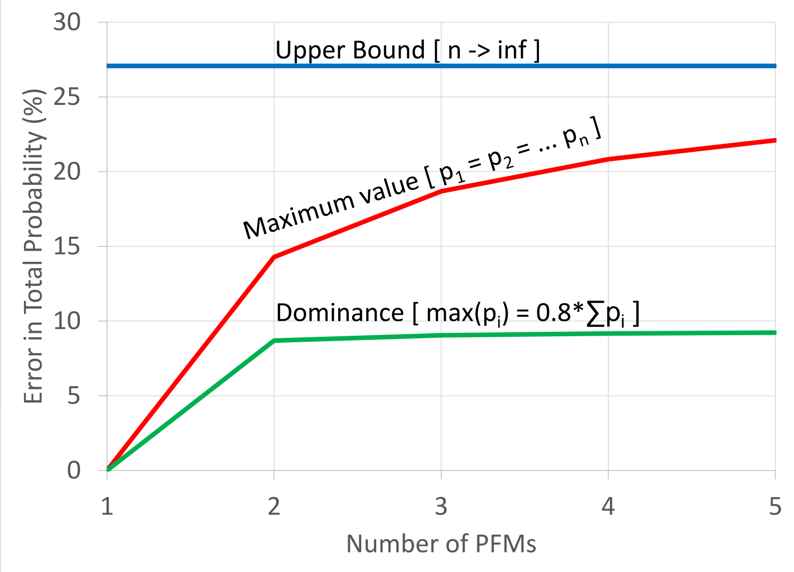
# Ignore Intersection Events and Their Probabilities

- Model PFMs as mutually exclusive
  - P(Total) = $\sum p_i$
- Reasonable approximation
  - $\sum p_i \approx 1 - \prod(1 - p_i)$
  - Common in dam and levee risk analysis
- When
  - $\sum p_i$ is small
  - $\max(p_i)$ is dominant

**The error will always be less than**




**Specific Example for $\sum p_i = 0.5$**



# Ignore Intersection Events and Distribute Their Probabilities

- Adjust probabilities so that
  - $\sum p_i' = 1 - \prod(1 - p_i)$
- Treat adjusted PFMs as mutually exclusive
  - P(Total) = $\sum p_i'$
- Hill, et al (2003) suggest a method for individual PFM adjustment

$$p_i' = p_i \frac{1 - \prod(1 - p_i)}{\sum p_i}$$

| PFM | Unadjusted $p_i$ | Adjusted $p_i'$ |
|---|---|---|
| 1 | 0.07 | 0.06 |
| 2 | 0.25 | 0.21 |
| 3 | 0.32 | 0.26 |

Example Calculation

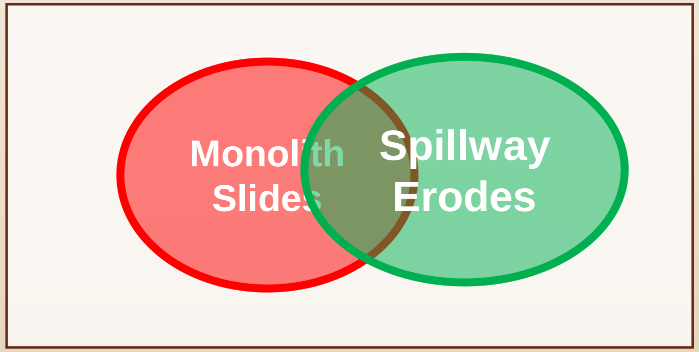
$$p_2' = 0.25 \frac{1-(1-0.07)(1-0.25)(1-0.32)}{0.07+0.25+0.32} = 0.21$$

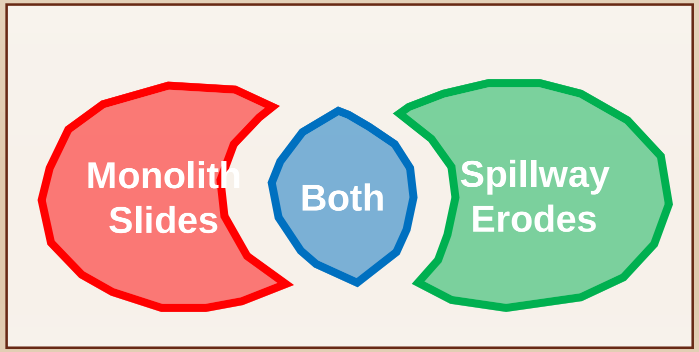
Verification

$$\sum p_i' = 0.06 + 0.21 + 0.26 = 0.53$$

$$1 - \prod(1 - p_i) = 1 - (1 - 0.07)(1 - 0.25)(1 - 0.32) = 0.53$$

# Enumerate Intersection Events and Their Probabilities



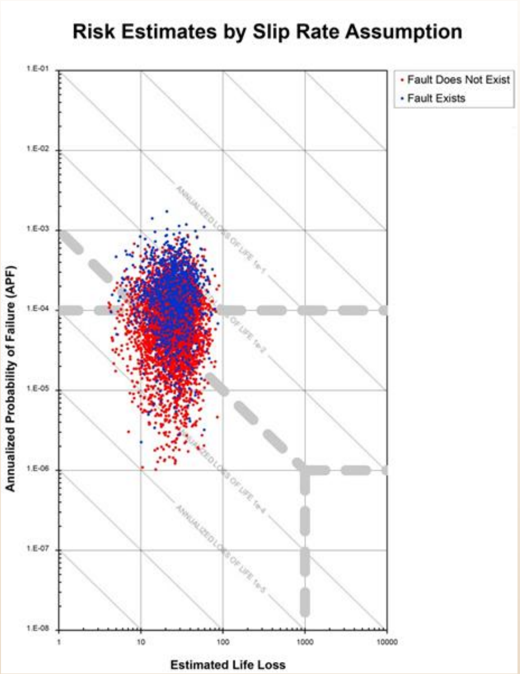

- For statistically independent events
- Mutually exclusive events and their probabilities
  - P(A notB) = P(A) [ 1 – P(B) ]
  - P(B notA) = P(B) [ 1 – P(A) ]
  - P(AB) = P(A)P(B)
- Since mutually exclusive, can be summed
  - P(Total) = P(A) [1–P(B)] + P(B) [1–P(A)] + P(A)P(B)
- Which can simplify to
  - P(Total) = P(A) + P(B) – P(A)P(B)
  - Recall this is the intersection equation
- How to attribute and portray the intersection event risks depends on both technical and policy considerations

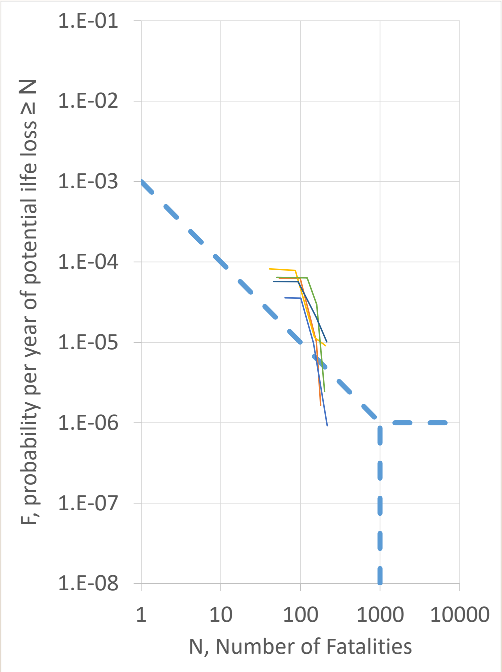
# Considerations

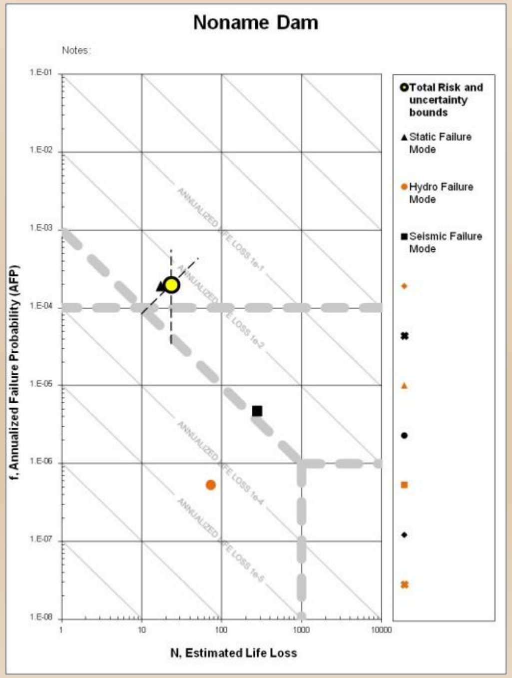
- Is the intersection small
- Is there a dominant PFM
- Impact on AFP and ALL estimates
- Potential impact on decision
- Consequence considerations (see chapter)

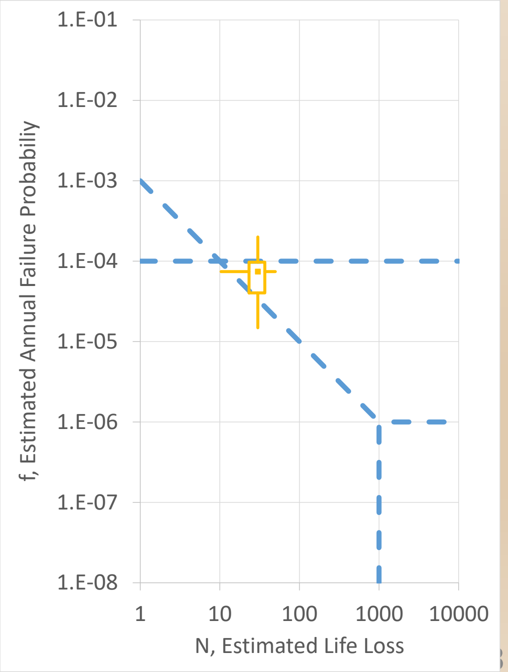
# Portraying Uncertainty

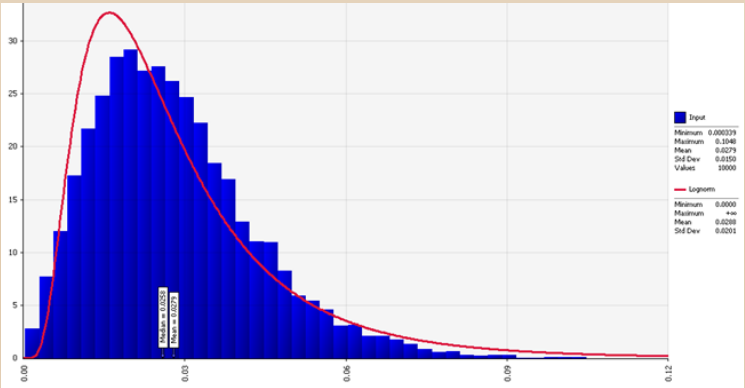
- Point cloud (f,N)
- Spaghetti plot (F,N)
- Confidence limits and intervals
- Whisker and Box Plots
- Many more options



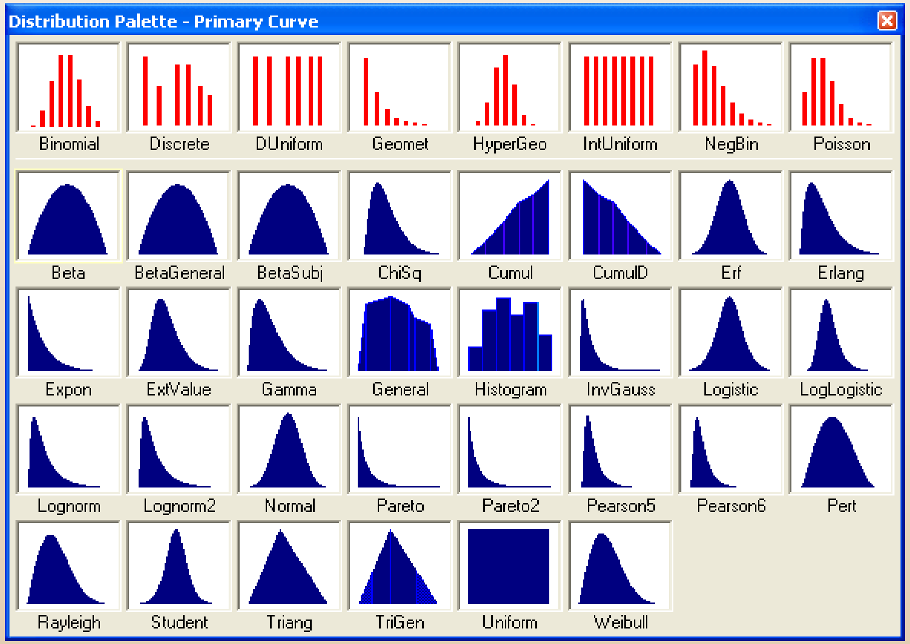



# Monte Carlo Simulation

- Used to evaluate output uncertainty
- When analytical solutions are difficult (or don't exist)
- An output distribution is built up over thousands of simulation trials
- Basic Steps:
    - Build a model or event tree
    - Assign probability distributions to the model inputs
    - Sample the model inputs based on their probability distributions
    - Record the output(s)
    - Evaluate the probability distributions of the model output(s)

# Selecting Input Distributions

- Does the input distribution really capture the uncertainty of the risk estimates or analysis results?
- Example: P(internal erosion initiates) = uniform (1E-3 to 2E-3) is probably too narrow given the uncertainty of the situation

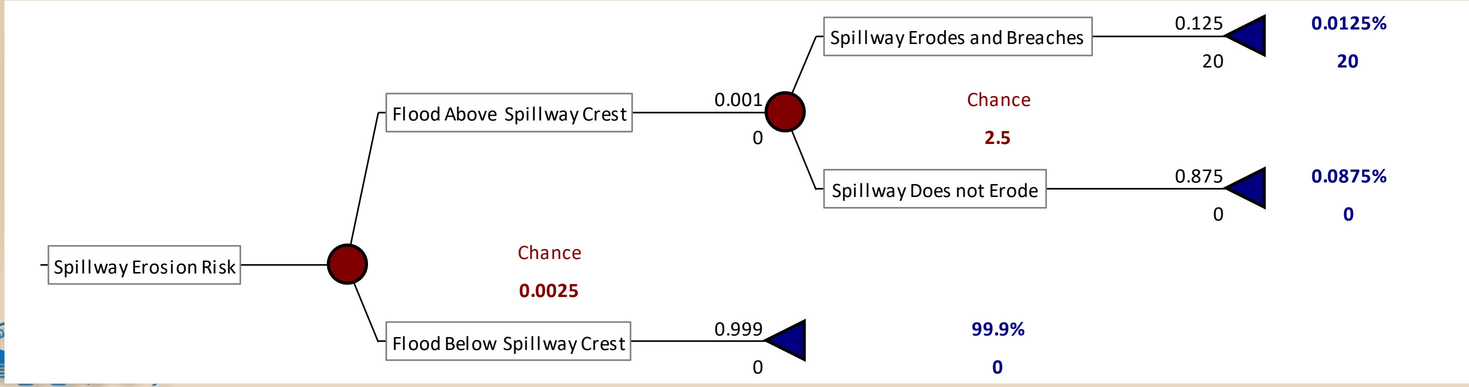


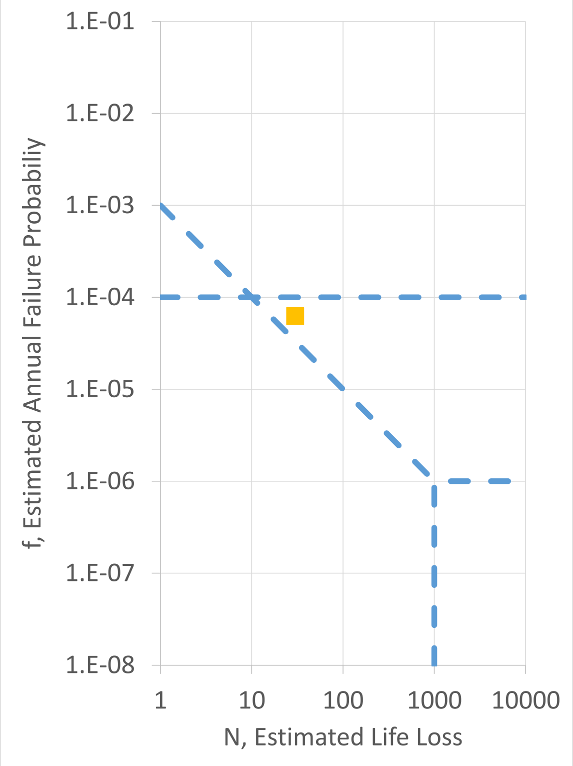

- But at the same time, be wary of distributions that span several orders of magnitude, since the mean can end up being skewed toward the upper bound unless care is taken

# Monte Carlo Example

- Estimate risk for a spillway erosion potential failure mode
- Step 1: Build a model
  - Best estimate probability of a flood above the spillway crest, 1/1000
  - Best estimate probability of spillway erosion leading to breach given the flood, 1/16
  - Best estimate life loss given breach, 30

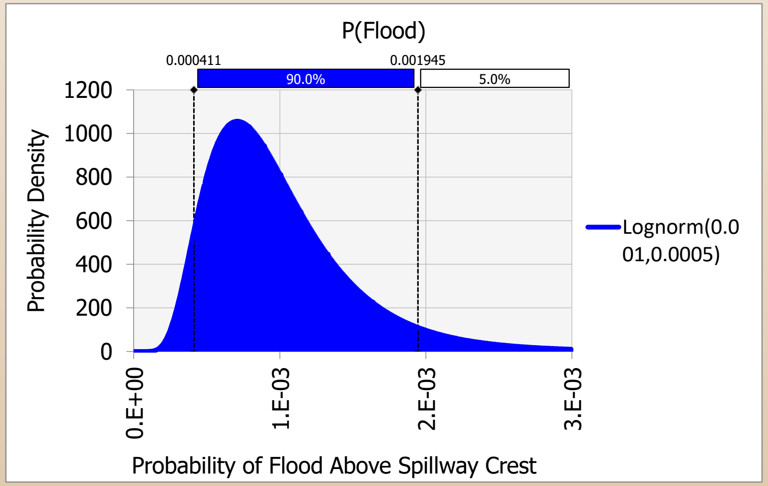
AFP = (1/1000)*(1/16) = 6E-5 ALL = (1/1000)*(1/16)*(30) = 2E-3

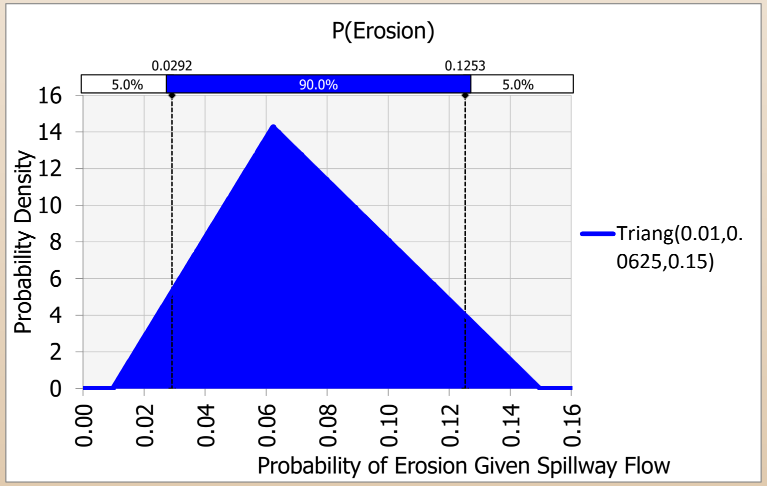


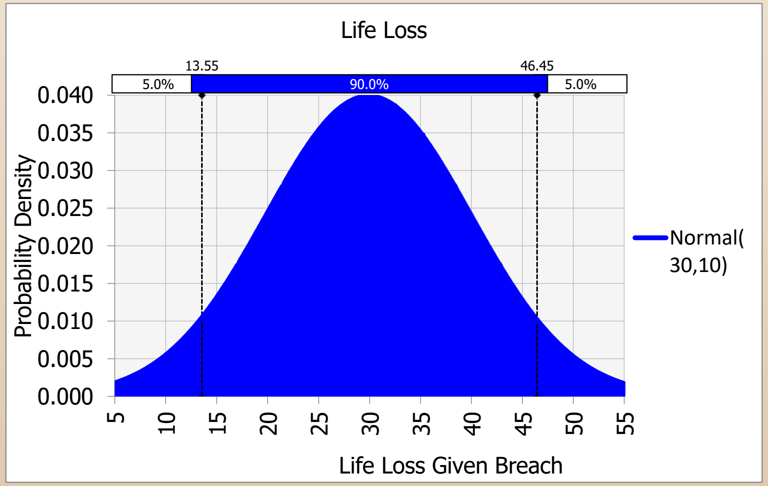



# Monte Carlo Example

- Step 2: Assign distributions to the model inputs



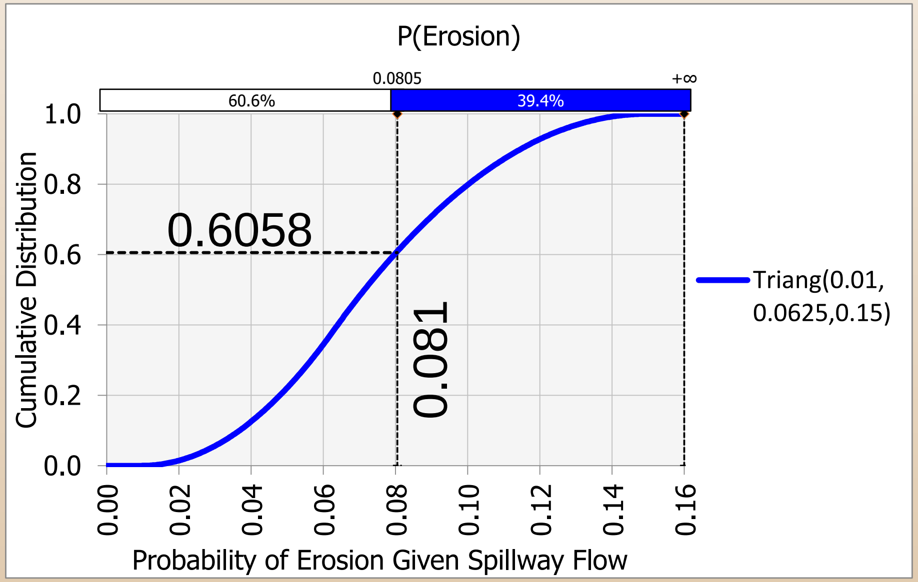



# Monte Carlo Example

- Step 3: Sample the inputs

| PRNG | P(Flood) | PRNG | P(Erosion) | PRNG | Life Loss |
|---|---|---|---|---|---|
| 0.04889 | 4.1E-4 | 0.8034 | 0.10 | 0.5351 | 30.9 |
| 0.1148 | 5.1E-4 | 0.6058 | 0.081 | 0.4089 | 27.7 |
| 0.5542 | 9.5E-4 | 0.8729 | 0.11 | 0.6163 | 33.0 |
| 0.8171 | 1.4E-3 | 0.4704 | 0.071 | 0.5503 | 31.3 |
| 0.0052 | 2.7E-4 | 0.4547 | 0.035 | 0.9555 | 46.5 |
| 0.2255 | 6.3E-4 | 0.2273 | 0.051 | 0.0598 | 14.4 |

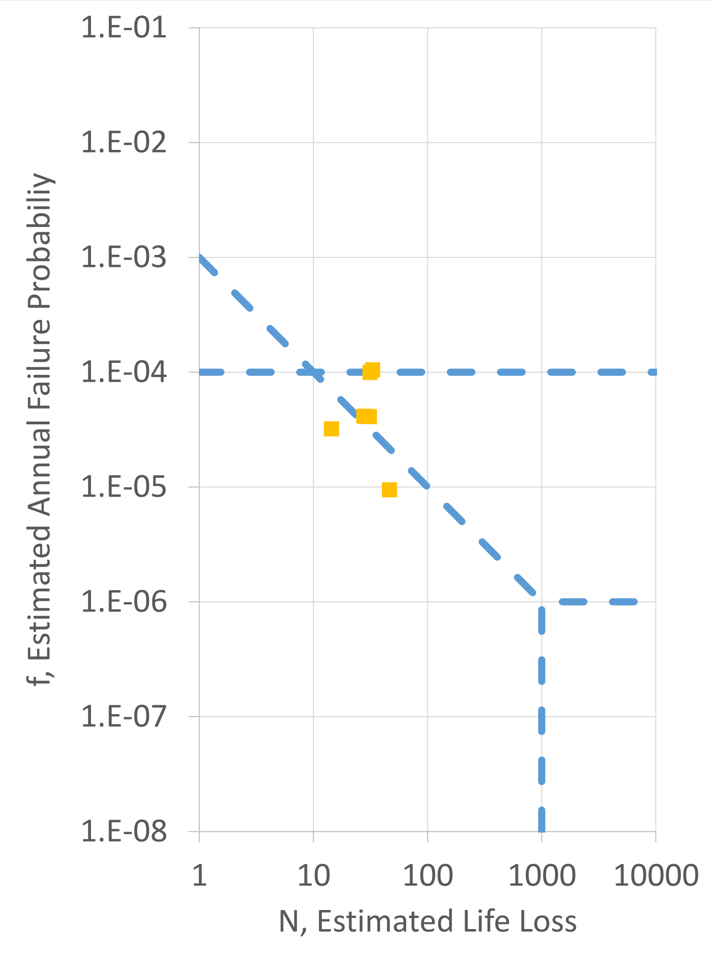



PRNG: Pseudo random number generator

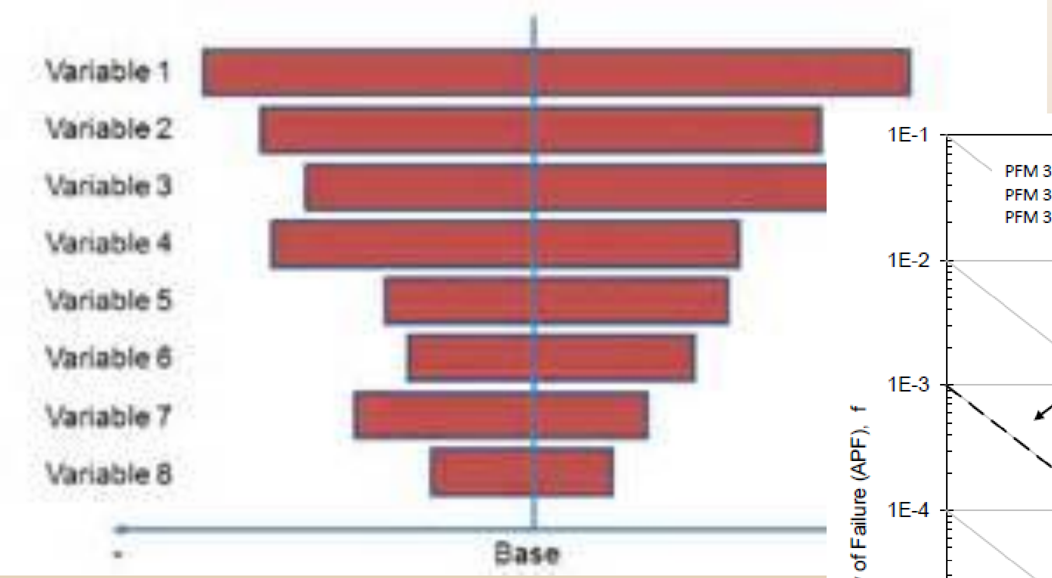
# Monte Carlo Example

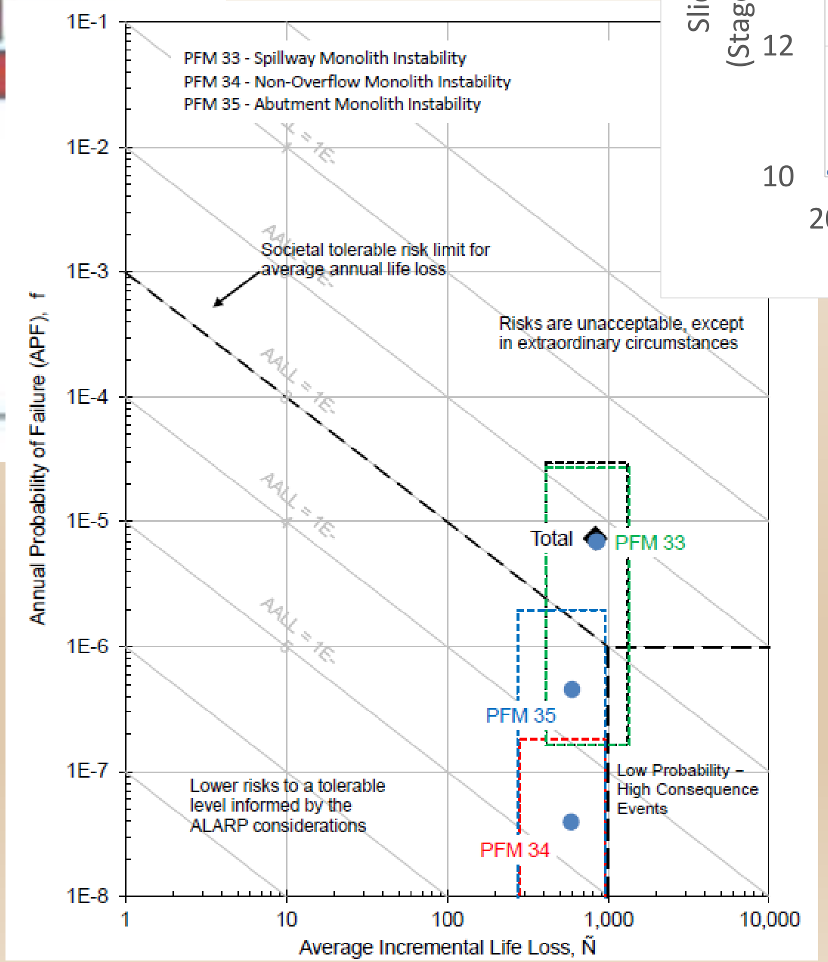
- Step 4: Compile the outputs

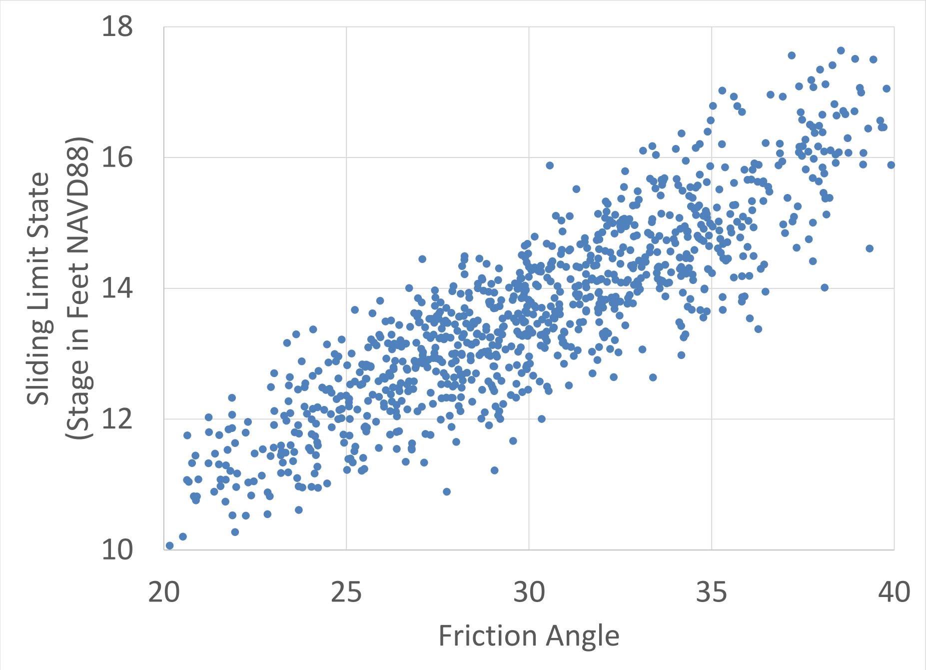
| P(Flood) | P(Erosion) | Life Loss | AFP | ALL |
|---|---|---|---|---|
| 4.1E-4 | 0.10 | 30.9 | 4.1E-5 | 1.3E-3 |
| 5.1E-4 | 0.081 | 27.7 | 4.1E-5 | 1.1E-3 |
| 9.5E-4 | 0.11 | 33.0 | 1.0E-4 | 3.4E-3 |
| 1.4E-3 | 0.071 | 31.3 | 9.9E-5 | 3.1E-3 |
| 2.7E-4 | 0.035 | 46.5 | 9.5E-6 | 4.4E-4 |
| 6.3E-4 | 0.051 | 14.4 | 3.2E-5 | 4.6E-4 |



# Portraying Sensitivity



# Takeaways

- The total risk of failure for a given facility is defined as the probability of the union of the individual PFMs.
- Adding the PFM risk estimates results in some over-counting of the intersection probabilities. In most cases, the error is small.
- In some cases, e.g., when the conditional failure probabilities are high, the error can be large enough to represent a quantifiable percentage of the total AFP. This does not always mean that an adjustment is required, but risk estimators should be aware.
- Understanding uncertainty is important because doing so can help guide the direction of future data collection or analysis.